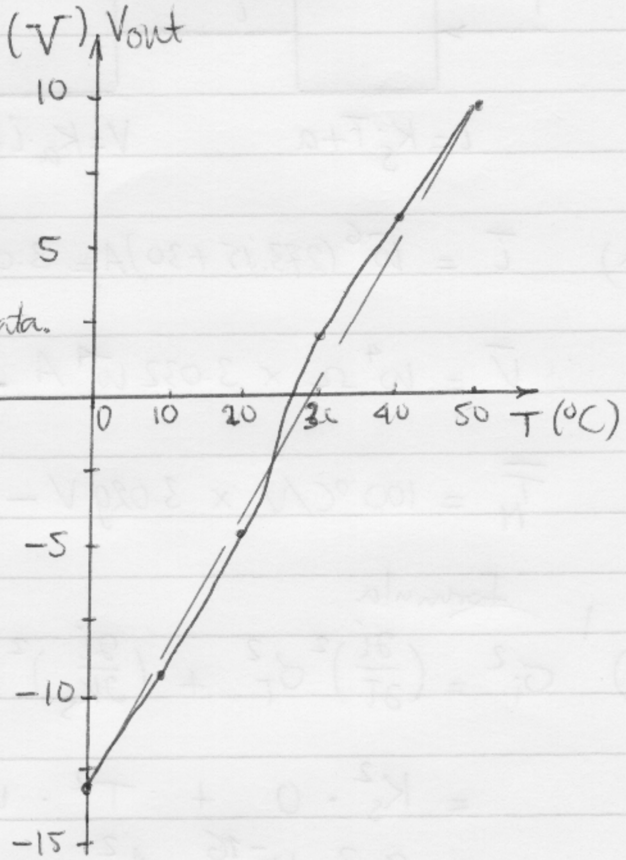
Midterm Jan 10, 2006 Solutions



1) a) the graph shows the tabulated date, the ideal straight line behavior, and (an estimate of)
2 the smooth response fitting the data.

b) $0 = a + K.I$

$V = a + K.T$

1 $a = V(T=0°C) = -12.84$

2 $$K = \frac{V(T=50) - V(T=0)}{50-0}$$

$$K = \frac{9.56 + 12.84}{50} \frac{V}{°C} = 0.448 \ V/°C$$

c)

T (°C)	0	10	20	30	40	50
V_{out} (V)	-12.84	-9.02	-4.82	2.19	6.03	9.56
V_{linear} (V)	-12.84	-8.36	-3.88	0.60	5.09	9.56
V_{nonlin} (V)	0	-0.66	-0.94	1.59	0.94	0

3 max. non-linearity occurs for $T \simeq 30°C$: $\hat{N} \simeq 1.6V$

$$\hat{N} = 1.6V \triangleq \frac{1.6V}{(9.56+12.84)V} = 0.07 = 7\%$$

d) • graph shows that $V_{out}(T=25°C) \simeq 0 \ V$

2 • linear approx : $V_{out}(25°C) \simeq V_{linear}(25°C) = -12.84 + 0.448 \cdot 25 V$

$= -1.64 \ V$

• Interpolation : $V_{out}(25°C) \simeq \frac{V_0(20°C) + V_0(30°C)}{2} = -1.32 \ V$

2| a). $O = a + K_I \cdot I_I + (K + K_M I_M) I + N(I)$

2 by definition I is the principal input value, in this case the pressure p; O is the output value, in this case the current i_{out}; I_I and I_M are the interfering and modifying inputs, respectively.

3 Since both offset and slope of the ideal straight line behavior (determined by the detector response at $p = 1$ bar and at $p = 6$ bar) are affected by the temperature, it is temperature that acts as both interfering and modifying input:

$$i_{out} = a + K_I \cdot \Delta T + (K + K_M \Delta T) \cdot p + N(p)$$

ideal straight line: $i_{out} = a + (K_I \Delta T) + (K + K_M \Delta T) p$:

$\underline{T = 25°C}$ $\quad i_{out} = 0.8 mA + \frac{16}{5} \frac{mA}{bar} \cdot p = (0.8 + 3.2 \cdot p)$ mA

$\underline{T = 30°C}$ $\quad i_{out} = \quad mA + \frac{18.5}{5} \frac{mA}{bar} \cdot p = (-0.2 + 3.7 \cdot p)$ mA

b) $T = 25°C \therefore \Delta T = 0$, $p = 1$ bar

3 $\Rightarrow \begin{cases} a = 4 \text{ mA} - (3.2 \text{ mA/bar}) \cdot 1 \text{ bar} = 0.8 \text{ mA} \\ K = 3.2 \text{ mA/bar} \end{cases}$

$T = 35°C \therefore \Delta T = +10°C$, $p = 1$ bar

$\Rightarrow \begin{cases} a + K_I \cdot 10 = -0.2 \text{ mA} \\ K + K_M \cdot 10 = 3.7 \text{ mA/bar} \end{cases} \Rightarrow \begin{cases} K_I = \frac{-0.2 - 0.8}{10} \frac{mA}{°C} \\ K_M = \frac{3.7 - 3.2}{10} \frac{mA}{bar\ °C} \end{cases}$

2 $\Rightarrow \begin{cases} K_I = -0.10 \text{ mA/°C} \\ K_M = +0.05 \text{ mA/(bar} \cdot \text{°C)} \end{cases}$

3)

$$T \longrightarrow \boxed{} \xrightarrow{\ i\ } \boxed{} \xrightarrow{\ V\ } \boxed{} \xrightarrow{T_M} T_M\,(^\circ C)$$

$i = K_S \cdot T + a \qquad V = K_a \cdot i + b \qquad T_M = K_D \cdot V + c$

a) $\bar{i} = 10^{-6}(273.15+30)A = 3.032\,10^{-4}A = 303.2\,\mu A$

1

$\bar{V} = 10^4\,\Omega \times 3.032\,10^{-4}A - 3\,10^{-3}V = 3.029\ V$

1

2 $\bar{T}_M = 100\,^\circ C/V \times 3.029\,V - 273\,^\circ C = 29.9\,^\circ C$ (Error $= -0.1\,^\circ C$)

1 formula

b) $\sigma_i^2 = \left(\frac{\partial i}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial i}{\partial K_S}\right)^2 \sigma_{K_S}^2 + \left(\frac{\partial i}{\partial a}\right)^2 \sigma_a^2$ $\qquad T = 30\,^\circ C = 303.15\ K$

1 $= K_S^2 \cdot 0 + T^2 \cdot 10^{-20} + 10^{-18}\ A^2$

$= 9.2\ 10^{-16}\ A^2$

Note: largest contribution, by far, comes from uncertainty in K_S!

$\sigma_V^2 = \left(\frac{\partial V}{\partial i}\right)^2 \cdot \sigma_i^2 + \left(\frac{\partial V}{\partial K_A}\right)^2 \sigma_{K_A}^2 + \left(\frac{\partial V}{\partial b}\right)^2 \sigma_b^2$

1 $= K_A^2 \cdot 9.2\,10^{-16} A^2 + \bar{i}^2\,\sigma_{K_A}^2 + 1 \cdot \sigma_b^2\ (V^2)$

$= 10^8\ 9.2\,10^{-16} + 9.2\,10^{-8} \cdot 900 + 10^{-6}\ V^2$

$= 9.2\,10^{-8} + 82.9\,10^{-6} + 10^{-6}\ V^2$

$= 83.9\,10^{-6}\ V^2$

Note: largest contribution now comes from uncertainty in K_A!

1 $\sigma_T^2 = \left(\frac{\partial T}{\partial V}\right)^2 \sigma_V^2 + \left(\frac{\partial T}{\partial K_D}\right)^2 \sigma_{K_D}^2 + \left(\frac{\partial T}{\partial c}\right)^2 \sigma_c^2$

$= K_D^2\ 83.9\,10^{-6} + \bar{V}^2 (0.1)^2 + 1 \cdot (0.1)^2\ (^\circ C)^2$

$= 83.9\,10^{-2} + 9.2\,10^{-2} + 10^{-2}\ (^\circ C)^2$

$= 94.1\,10^{-2}\ ^\circ C^2$

2 $\Rightarrow \sigma_T = 0.97\,^\circ C$ With largest contribution from σ_V and thus K_A.

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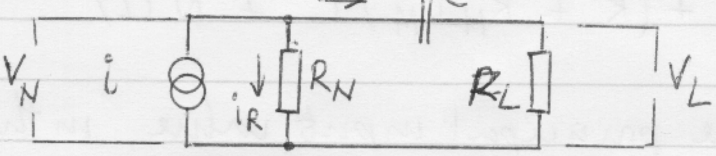
a)



• $V_N = i_R \cdot R_N = i_C (Z_C + R_L)$

$\Rightarrow \quad i_C = i_R \cdot \dfrac{R_N}{Z_C + R_L}$

• $i = i_R + i_C = i_R + i_R \dfrac{R_N}{Z_C + R_L} = i_R \left(\dfrac{R_N + R_L + Z_C}{Z_C + R_L} \right)$

• $V_L = i_C \cdot R_L = i_R \, R_L \dfrac{R_N}{Z_C + R_L}$

$= i \dfrac{Z_C + R_L}{R_N + R_L + Z_C} \cdot R_L \cdot \dfrac{R_N}{Z_C + R_L} = i \dfrac{R_N \cdot R_L}{R_N + R_L + Z_C}$

2 $\quad = i \cdot \dfrac{R_N R_L}{(R_N + R_L) + \frac{1}{C \cdot s}} = i \cdot \dfrac{R_N R_L C \cdot s}{(R_N + R_L) C \cdot s + 1}$

b) 2 $\quad \Rightarrow G(s) = \dfrac{\Delta \tilde{V}_L(s)}{\Delta \tilde{i}(s)} = \dfrac{k \cdot s}{1 + \tau \cdot s}$ with $\begin{cases} k = R_N R_L C = 10^4 \; \Omega \sec \\ \tau = (R_N + R_L) C = 0.2 \sec \end{cases}$

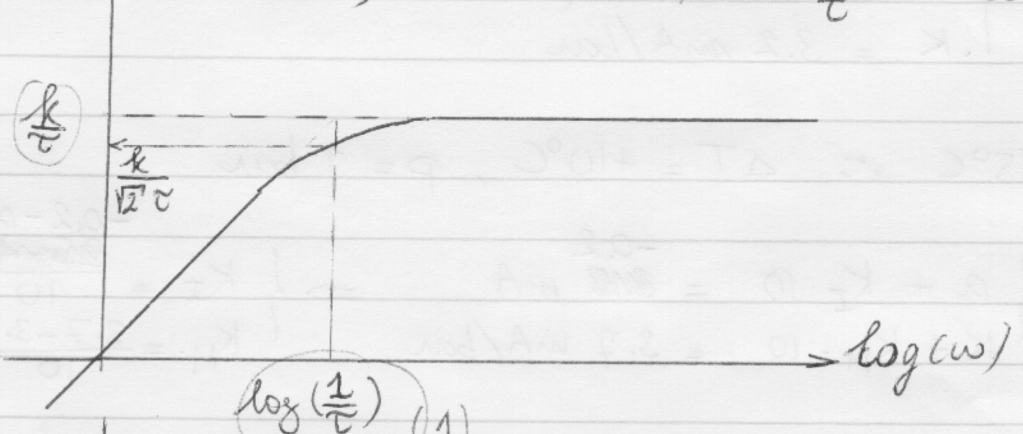
2 c) 1^{st} order system (e.g: max phase shift is $90°$).

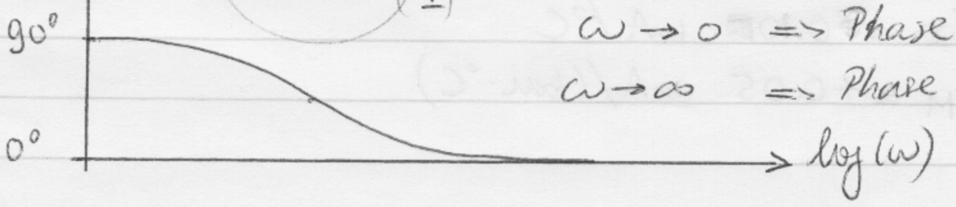
d) $AR(\omega) = |G(j\omega)| = \left| \dfrac{k \cdot j\omega}{1 + \tau j\omega} \right| = \dfrac{k\omega}{\sqrt{1 + \omega^2 \tau^2}}$

e)

2 $\quad \omega \to 0 \Rightarrow AR \to 0 \quad ; \quad \omega \to \infty \Rightarrow AR \to \dfrac{k}{\tau} \qquad \omega = \dfrac{1}{\tau} \Rightarrow AR = \dfrac{k}{\tau\sqrt{2}}$

2





$\omega \to 0 \Rightarrow$ Phase $\to 90° = \frac{\pi}{2}$ rad

$\omega \to \infty \Rightarrow$ Phase $\to 0° = 0$ rad